**Mathematical Methods in Earth Sciences**

Lecture 1 - Mar/03/19

Review of trigonometry and basic calculus

*Trigonometry*

**When is it useful?** Everywhere! Anything involving coordinate systems (e.g. maps, orbits), resolving of forces, angular variations (e.g. statistics), etc.

Given the triangle



We define

**What do they look like?**

**Note that** cos() = cos(−) (“even” function) and sin() = − sin(−) (“odd” function).

In 2D, Pythagoras’ theorem tells us that

*h*2=*o*2+*a*2

Dividing by the square of the hypotenuse (*h*2) we get

**Note that** cos2 is the same thing as (cos )2

**Example 1** Coordinate transformations



**Example 2** Calculate orbital distance for inferior planets



The angle is expressed in *radians* (which are unitless). When working in radians, it is simple to calculate the arc legnth *l* subtended by the angle :

*l = r*

where *r* is the radius of the circle.



Since the perimeter of a circle is 2*r*,then “360◦”= 2radians.You sould always use radians when doing math problem; degress are for maps.

**Useful approximations** When  is small ( ≈ 0) we have

sin  ≈

cos  ≈ 1 - 1

tan  ≈

These formulae *only* work if  is in radians.

**Example**

Trigonometric Parallax, Stellar Parallax :

 

Useful formulas include:

1. Double angle formulae (*where does this come from?*):

sin( + ) = sin cos + cos sin

cos( + ) = cos cos − sin sin

You can also obtain sin and cos of ( – ) by replacing with – in the above formulae. You can also obtain sin and cos of 2 by setting  = .

For the following general triangle we also have:



1. Law of Cosines

*a*2=*b*2+*c*2-2*bc* cos

*b*2=

*c*2=

1. Law of Sines

*Differentiation*

**When is it useful?** Anywhere that we are interested in changing quantities (ﬂuid ﬂux, topographic gradients, stock markets etc. etc.); also for ﬁnding maxima/minima.

We will denote any function of *x*, for instance sin(*x*), *x*2, etc., as *f*(*x*). It is important to understand that if *f*(*x*) = *x*2, then *f*(*x*+*h*) = (*x*+*h*)2. Likewise, if *f*(*x*) = cos(*x*), then *f*(*x*+*h*) =cos(*x* + *h*) and so on.

When we take the slope of a line (i.e. its gradient) between two points (*x*1,*y*1) and (*x*2,*y*2) we calculate it by using

If the *y* values are described by some function of *x*, *y* = *f*(*x*), then we can rewrite the slope of the line as

Incidentally, this makes it obvious that

Now let’s instead write *x*2 = *x*1 + *h*. Then as *h* becomes vanishingly small, we are calculating the slope of the line at position x1. The slope of the line described by *y* = *f*(*x*) at position *x* is known as the *derivative* of *f*(*x*) with respect to *x* and is written or *f’*(*x*). Formally, we write

**What does this mean?** Example: population change

|  |  |
| --- | --- |
| *t*(Year) | Population |
| 1980 | 9,847 |
| 1982 | 9,856 |
| 1984 | 9,855 |
| 1986 | 9,862 |
| 1988 | 9,884 |
| 1990 | 9,962 |
| 1992 | 10,036 |
| 1994 | 10,109 |
| 1996 | 10,152 |
| 1998 | 10,175 |
| 2000 | 10,186 |

The basic rules of differentiation are:

1. If *f*(*x*)=constant then =
2. If *f*(*x*) = *xn* then =
3. If *F*(*x*) = *f*(*x*) + *g*(*x*) then =
4. If *f*(*x*) = *u*(*x*)*v*(*x*) then = (product rule)
5. If *f*(*x*) = *g*(*u*(*x*)) then = (chain rule)

Special functions that you’ll need to know are

if *f*(*x*)=ln(*x*) then =

if *f*(*x*)=*eax* then =

if *f*(*x*)=sin(*x*) then =

if *f*(*x*)=cos(*x*) then =

To ﬁnd the maximum/minimum of a function we ﬁnd the value(s) of *x* at which

=0

If >0 on an interval, then *f* is increasing on that interval.

If <0 on an interval, then *f* is decreasing on that interval.

To determine whether these points are maxima or minima, we diﬀerentiate a second time:

if then minimum

if then maximum

**Example**Newton-Raphson method, Starting with *x*1=2, find the third approximation *x*3 to the root of the equation

*x*3-2*x*-5=0

**Application**Find Lagrangian points using Newton-Raphson method

**





*Integration*

**Where is it useful?** Gravity, moment of inertia, energy . . . see examples below.

You can think of integration as the reverse of diﬀerentiation. Physically, diﬀerentiation involves taking the slope of a line, while integration involves totalling up the area under a curve.

Diﬀerentiation involves loss of information e.g. diﬀerentiating *f*(*x*) and *f*(*x*)+*c* will give the same answer (*f*(*x*)). So when we integrate, we need to add in an undetermined constant of integration:

where *c* is a constant. This is an *indefinite integral* because the limits of integration are not deﬁned. If they are deﬁned, then instead we get

**Simple integration examples**

1. If *f*(*x*) = *xn* then
2. If *f*(*x*) = cos(2*x*) then
3. If *f*(*x*) = *eax* then
4. If *f*(*x*) = then

Some examples of when integration is useful:

1. Area beneath a curve *f*(*x*) between *x* = *a* and *x* = *b* is

Area=

Note that this expression has the correct units for area.

1. The average (mean) value of *f*(*x*) between *x* = *a* and *x* = *b* is

Mean =

Again, note that this expression has the correct units.

1. Recall that work = force times distance, or *dW* = *f*(*x*)*dx* where *f*(*x*) is the force, *dx* is the increment in distance and *dW* is the increment in work. The work done in moving from *a* to *b* is
2. The last thing to remember is *integration by parts*. It is easiest to demonstrate how this works with an example:.

To show where this comes from, we start with the chain rule

*d*(*uv*) = *u dv* + *v du*

Integrating both sides yields

or

Always choose *u* so that if you diﬀerentiate it enough times it will become a constant. Remember that *you can always check your answer by differentiating!*

**Second example**: *ex* sin *x*

**Third example:** Find the area enclosed by the ellipse

